Ex.

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 10 \end{bmatrix}$$

$$|A| = 2$$

$$A^{-1} = \frac{1}{2}\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$$

$$= \frac{1}{2}\begin{bmatrix} 10 & -2 \\ -4 & 1 \end{bmatrix}^T$$

$$= \frac{1}{2}\begin{bmatrix} 10 & -4 \\ -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -2 \\ -1 & 1/2 \end{bmatrix}$$

Ex.

$$A = \begin{bmatrix} 2 & 2 & 0 \\ -2 & 1 & 1 \\ 3 & 0 & 1 \end{bmatrix} \qquad |A| = 12$$

$$c_{11} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = 1 \qquad C_{21} = -\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} = -2$$

$$c_{12} = -\begin{bmatrix} -2 & 1 \\ 3 & 1 \end{bmatrix} = 5 \qquad C_{22} = \begin{bmatrix} 2 & 0 \\ 3 & 1 \end{bmatrix} = 2$$

$$C_{13} = \begin{bmatrix} -2 & 1 \\ 3 & 0 \end{bmatrix} = -3 \qquad C_{23} = -\begin{bmatrix} 2 & 2 \\ 3 & 0 \end{bmatrix} = 6$$

$$C_{31} = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} = 2$$

$$C_{32} = -\begin{bmatrix} 2 & 0 \\ -2 & 1 \end{bmatrix} = -2 \qquad C = \begin{bmatrix} 1 & 5 & -3 \\ -2 & 2 & 6 \\ 2 & -2 & 6 \end{bmatrix}^T$$

$$C_{33} = \begin{bmatrix} 2 & 2 \\ -2 & 1 \end{bmatrix} = 6$$

$$C = \begin{bmatrix} 1 & -2 & 2 \\ 5 & 2 & -2 \\ -3 & 6 & 6 \end{bmatrix}$$

$$A^{-1} = \frac{1}{12}\begin{bmatrix} 1 & -2 & 2 \\ 5 & 2 & -2 \\ -3 & 6 & 6 \end{bmatrix} = \begin{bmatrix} 1/12 & -1/6 & 1/6 \\ 5/12 & 1/6 & -1/6 \\ -1/4 & 1/2 & 1/2 \end{bmatrix}$$

ROW OPPERATING METHOD FOR FINDING INVERSE OF A MATRIX.

this method is particularly useful for larger matrices. If we argument $\underline{A}$ and $\underline{I}$, we find A^{-1} from successive row operations. It is best illistrated by an example.

ex. find A^{-1}

$$A = \begin{bmatrix} 2 & 0 & 1 \\ -2 & 3 & 4 \\ -5 & 5 & 6 \end{bmatrix}$$

argument with $\underline{I}$.

$$[A|I] = \left[\begin{array}{ccc|ccc} 2 & 0 & 1 & 1 & 0 & 0 \\ -2 & 3 & 4 & 0 & 1 & 0 \\ -5 & 5 & 6 & 0 & 0 & 1 \end{array}\right] \quad \tfrac{1}{2} R_1$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 1/2 & 1/2 & 0 & 0 \\ -2 & 3 & 4 & 0 & 1 & 0 \\ -5 & 5 & 6 & 0 & 0 & 1 \end{array}\right] \quad \begin{array}{l} 2R_1 + R_2 \\ \\ 5R_1 + R_3 \end{array}$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 1/2 & 1/2 & 0 & 0 \\ 0 & 3 & 5 & 1 & 1 & 0 \\ 0 & 5 & 17/2 & 5/2 & 0 & 1 \end{array}\right] \quad \begin{array}{l} 1/3\ R_2 \\ \\ 1/5\ R_3 \end{array}$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 1/2 & 1/2 & 0 & 0 \\ 0 & 1 & 5/3 & 1/3 & 1/3 & 0 \\ 0 & 1 & 17/10 & 5/10 & 0 & 1/5 \end{array}\right] \quad -R_2 + R_3$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 1/2 & 1/2 & 0 & 0 \\ 0 & 1 & 5/3 & 1/3 & 1/3 & 0 \\ 0 & 0 & 1/30 & 1/6 & 1/3 & 1/5 \end{array}\right] \quad 30R_3$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 1/2 & 1/2 & 0 & 0 \\ 0 & 1 & 5/3 & 1/3 & 1/3 & 0 \\ 0 & 0 & 1 & 5 & 10 & 6 \end{array}\right] \quad \begin{array}{l} -5/3 R_3 + R_2 \\ -\frac{1}{2} R_3 + R_1 \end{array}$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & -5 & -3 \\ 0 & 1 & 0 & -8 & 17 & -10 \\ 0 & 0 & 1 & 5 & 16 & 6 \end{array}\right]$$

$$\underbrace{\qquad\qquad\qquad}_{A^{-1}}$$

EX.

$$2x_1 - 9x_2 = 15$$

$$5x_1 + 6x_2 = 16$$

SOLUTION:

put the above in matrix compact for.

$$\begin{bmatrix} 2 & -9 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 15 \\ 16 \end{bmatrix}$$

$$\underline{A}\,\underline{x} = \underline{b}$$

$$\underline{A}^{-1}\,\underline{A}\,\underline{x} = \underline{A}^{-1}\,\underline{b} \qquad b/c \quad \underline{A}^{-1}A = I$$

$$\underline{I}\,\underline{x} = \underline{A}^{-1}\,\underline{b}$$

$$\therefore \quad \underline{x} = \underline{A}^{-1}\,\underline{b}$$

$$\det A = |\underline{A}| = \begin{bmatrix} 2 & -9 \\ 3 & 6 \end{bmatrix} = (2)(6) - (-9)(3) = 39 \neq 0$$

Ø would mean that there is 2 rows or two columns that are the same. $\therefore$ A^{-1} exists

$$[\text{cofactors of } A]^T$$

$$\begin{bmatrix} 6 & -3 \\ 9 & 2 \end{bmatrix}^T = \begin{bmatrix} 6 & 9 \\ -3 & 2 \end{bmatrix} = \text{Adj } \underline{A}$$

$$A^{-1} = \frac{1}{|A|}\left[\text{adj } A\right]$$

$$A^{-1} = \frac{1}{39}\begin{bmatrix} 6 & 9 \\ -3 & 2 \end{bmatrix}$$

$$\underline{x} = \frac{1}{39}\begin{bmatrix} 6 & 9 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 15 \\ 16 \end{bmatrix} = \begin{bmatrix} 6 \\ -\frac{1}{3} \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

THE EIGEN VALUE PROBLEM.

A square matrix $\underline{A}$ is said to have numbers known as eigen values, and none zero eigen values such that

$$\underline{A}\,\underline{k} = \lambda\,\underline{k}$$

square matrix → $\underline{A}$

eigen value (scaller (complex)) → λ

eigenvector → $\underline{k}$

Ex. Verify that $\underline{k} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

is the eigenvector of the matrix

$$\underline{A} = \begin{vmatrix} 0 & -1 & -3 \\ 2 & 3 & 3 \\ -2 & 1 & 1 \end{vmatrix}$$

$$\underline{A}\,\underline{k} = \underbrace{\begin{vmatrix} 0 & -1 & -3 \\ 2 & 3 & 3 \\ -2 & 1 & 1 \end{vmatrix}}_{\underline{A}} \underbrace{\begin{vmatrix} 1 \\ -1 \\ 1 \end{vmatrix}}_{\underline{k}} = \underbrace{\begin{vmatrix} -2 \\ 2 \\ -2 \end{vmatrix}}_{\lambda \underline{k}} = \underbrace{-2}_{\lambda} \underbrace{\begin{vmatrix} 1 \\ -1 \\ 1 \end{vmatrix}}_{\underline{k}}$$

so from the definition, we can write

$$\underline{A}\,\underline{k} - \lambda\,\underline{k} = \underline{0}$$
$$\left[\underline{A} - \lambda\,\underline{I}\right]\underline{k} = \underline{0}$$

$\underline{I}$: unit matrix $(n \times n)$
$\underline{A} = n \times n$ square matrix
$\underline{k}$: $1 \times n$ vector matrix

then the set of equations is

$$a_{11}(a_{11} - \lambda)k_1 + \ldots\ldots\ldots a_{1n}k_n = \underline{0}$$
$$a_{21}k_1 + a_{22}(a_{22} - \lambda)k_2 \ldots a_{2n}k_n = 0$$

$$a_{n1}k_1 + \ldots\ldots\ldots + a_{nn}(a_{nn} - \lambda) = 0$$

to find the eigen values & eigen vectors, we solve the determinate $|\underline{A} - \lambda\,\underline{I}| = \underline{0}$

EX. $\underline{A} = \begin{vmatrix} 3 & 4 \\ -1 & 7 \end{vmatrix}$

$$|\underline{A} - \lambda \underline{I}| = 0$$

$$\begin{vmatrix} (3-\lambda) & 4 \\ -1 & (7-\lambda) \end{vmatrix} = 0$$

$$(7-\lambda)(3-\lambda) - (-1)(4) = 0$$

$$(\lambda - 5)^2 = 0 \qquad \lambda = 5$$

note: for a 2x2 matrix, there will be 2 eigen values, a 3x3 will have 3 eigen values and so on.

to find the eigen vectors for the above example

$$[\underline{A} - \lambda \underline{I}]\, \underline{k} = 0$$

$$\left(\begin{vmatrix} 3 & 4 \\ 7 & -1 \end{vmatrix} - 5 \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \right) \begin{vmatrix} k_1 \\ k_2 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \end{vmatrix}$$

$$\begin{bmatrix} -2 & 4 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = 0$$

or

$$-2k_1 + 4k_2 = 0$$
$$-k_1 + 2k_2 = 0$$

b/c eqn 1, 2 are the same we choose

$\underline{k} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

in this case, since $\lambda_1 = \lambda_2$ the the eigen vector of $\underline{A}$ which are $k_1 = k_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

<u>Check it</u>

$$\underline{A}\,\underline{k_1} = \lambda_1 \underline{k_1}$$

$$\begin{bmatrix} 3 & 4 \\ -1 & 7 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \lambda \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$\begin{bmatrix}10\\5\end{bmatrix} = \lambda\begin{bmatrix}2\\1\end{bmatrix}$ $\quad \underline{\underline{\lambda = 5}}$

note: you will come across eigen values and eigenvectors for example in control engineering.

In statistics where the condition of a matrix $\underline{A}$ is very important.

EX. find the eigen values and eigen vectors of $\underline{A}$

$$\underline{A} = \begin{bmatrix}6 & -1\\5 & 4\end{bmatrix}$$

$$|\underline{A} - \lambda\underline{I}| = 0$$

$$\begin{bmatrix}(6-\lambda) & -1\\5 & (4-\lambda)\end{bmatrix} = |\underline{A} - \lambda\underline{I}| = 0$$

$$(6-\lambda)(4-\lambda) - (-1)(5) = 0$$

$$\lambda^2 - 10\lambda + 29 = 0$$

$$\lambda_1 = 5 + 2i \qquad \lambda_2 = 5 - 2i$$

let λ_1 be the eigenvalue of the eigen vector k_1

$$[\underline{A} - \lambda_1\underline{I}]k_1 = 0$$

$$\left(\begin{bmatrix}6 & -1\\5 & 4\end{bmatrix} - (5+2i)\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\right)\begin{bmatrix}k_1\\k_2\end{bmatrix} = \underline{0}$$

$$\left(\begin{bmatrix}6 & -1\\5 & 4\end{bmatrix} - \begin{bmatrix}5+2i & 0\\0 & 5+2i\end{bmatrix}\right)\begin{bmatrix}k_1\\k_2\end{bmatrix} = \underline{0}$$

$$\begin{bmatrix}1-2i & -1\\5 & -1-2i\end{bmatrix}\begin{bmatrix}k_1\\k_2\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

$(1-2i)k_1 - k_2 = 0$ (1)
$5k_1 - (1+2i)k_2 = 0$ (2)
} are they the same?

rewrite (2) as

$$\frac{5}{1+2i}k_1 = k_2$$

hence they are the same, (1) & (2) are identical

we choose $k_1 = 1$, and we will find $k_2 = 1-2i$

$k_1 = \begin{bmatrix} 1 \\ 1-2i \end{bmatrix}$ this the eigen vector

associated $\bar{c}$ the eigen value.

In a similar manner, we find that k_2 associated with $\lambda_2 = 5-2i$

$$\left(\begin{bmatrix} 6 & -1 \\ 4 & 7 \end{bmatrix} - (5-2i)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)\begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$(1+2i)k_1' - k_2' = 0$ (3)

$5k_1' - (1-2i)k_2' = 0$ (4)

rewrite (4) as

$$5 \cdot k_1' = (1-2i)k_2'$$

$$\frac{5k_1'}{1-2i} = k_2'$$

$$\frac{5(1+2i)}{1^2+2^2} = k_2'$$

$$k_2' = (1+2i)$$

therefore we see that eqn (3) & (4) are the same.

we choose $k_1' = 1$

$$k'_2 = (1+2i)$$

$$\underline{k}' = \begin{bmatrix} 1 \\ 1+2i \end{bmatrix}$$

EX. find the eigen values and vectors of

$$\underline{A} = \begin{bmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

SA: $|\underline{A} - \lambda \underline{I}| = 0 = \begin{bmatrix} (1-\lambda) & 2 & 1 \\ 6 & (-1-\lambda) & 0 \\ -1 & -2 & (-1-\lambda) \end{bmatrix}$

and we get the equations

$$-\lambda^3 - \lambda^2 + 12\lambda = 0$$

$$\lambda(\lambda+4)(\lambda-3) = 0 \quad \lambda_1 = 0 \quad \lambda_2 = -4 \quad \lambda_3 = 3$$

$\underline{\lambda_1 = 0}$ we may use the guass jordon method

$= [\underline{A} - \lambda \underline{I} \mid \underline{0}] \Rightarrow$ augmented matrix

$$= \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 6 & -1 & 0 & 0 \\ -1 & -2 & -1 & 0 \end{array}\right] \begin{array}{l} -6R_1 + R_2 \\ \\ R_1 + R_3 \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & -13 & -6 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad -\frac{R_2}{13}$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 1 & 6/13 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad -2R_2 + R_1$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 1/13 & 0 \\ 0 & 1 & 6/13 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

from the last result we may write

$$k_1 + 0k_2 + \frac{1}{13}k_3 = 0$$

$$k_2 + 6/13\, k_3 = 0$$

$$\therefore\ k_1 = -\frac{1}{13}k_2 \quad \& \quad k_2 = -\frac{6}{13}k_3$$

choose $\underline{k}_3 = -13$

$$\underline{k}_1 = \begin{bmatrix} 1 \\ 6 \\ -13 \end{bmatrix}$$

$\lambda_2 = -4$

$$[\underline{A} - \lambda_2 \underline{\underline{I}} \,|\, 0]$$

$$\left[\begin{array}{ccc|c} 5 & 2 & 1 & 0 \\ 6 & 3 & 0 & 0 \\ -1 & -2 & 3 & 0 \end{array}\right] \xRightarrow{-R_3} \left[\begin{array}{ccc|c} 5 & 2 & 1 & 0 \\ 6 & 3 & 0 & 0 \\ 1 & 2 & -3 & 0 \end{array}\right] \quad \text{switch } R_1 \,\&\, R_3$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & -3 & 0 \\ 6 & 3 & 0 & 0 \\ 5 & 2 & 1 & 0 \end{array}\right] \begin{array}{l} -6R_1 + R_2 \\ \\ -5R_1 + R_3 \end{array} \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\therefore\ k_1 + k_3 = 0 \quad \text{or} \quad k_1 = -k_3$$

$$k_2 - 2k_3 = 0 \quad \text{or} \quad k_2 = +2k_3$$

we choose $k_3 = 1$

$$\underline{k}_2 = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$$

$\lambda_3 = 3$

$$[\underline{A} - 3\underline{\underline{I}} \,|\, 0] = \left[\begin{array}{ccc|c} -2 & 2 & 1 & 0 \\ 6 & -4 & 0 & 0 \\ -1 & -2 & -4 & 0 \end{array}\right]$$

after row opperations.

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 3/2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\therefore$ $k_1 = -k_3$ choose $k_3 = -2$

$k_2 = -3/2\, k_3$

$$\therefore \quad \underline{k}_3 = \begin{bmatrix} 2 \\ 3 \\ -2 \end{bmatrix}$$

SYMETRIC MATRIX.

If $\underline{A}$ is an $(n \times n)$ matrix such that: $A^T = A$, then you have a symetrical matrix.

ORTHOGONAL MATRIX.

If $\underline{A}$ is an $(n \times n)$ symetric matrix and $\underline{A}^{-1} = \underline{A}^T$, the matrix is said to be orthogonal matrix and it follows that

$$\underline{A}^T \underline{A} = \underline{A}^{-1} \underline{A} = \underline{I}$$

DIAGONALIZATION.

Let $\underline{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ $\underline{B} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$

$$\underline{A}\underline{B} = \begin{bmatrix} (a_{11}b_{11} + a_{12}b_{21}) & (a_{11}b_{12} + a_{12}b_{22}) \\ (a_{21}b_{11} + a_{21}b_{21}) & (a_{21}b_{12} + a_{22}b_{22}) \end{bmatrix}$$

Let us write $\underline{B} = [\underline{x}_1 \;\; \underline{x}_2]$

$$\underline{x}_1 = \begin{pmatrix} b_{11} \\ b_{21} \end{pmatrix} \quad \underline{x}_2 = \begin{pmatrix} b_{12} \\ b_{22} \end{pmatrix}$$

then

$$\underline{A}\underline{B} = \underline{A}[\underline{x}_1 \;\; \underline{x}_2] = \underline{A}\underline{x}_1 + \underline{A}\underline{x}_2$$

what the objective hear is to find an $(n \times n)$ matrix $\underline{P}$ such that

$$\underline{P}^{-1}\underline{A}\,\underline{P} = \underline{D}$$

multiply by P.

square

diagonal matrix.

$$\underline{P}\,\underline{P}^{-1}\underline{A}\,\underline{P} = \underline{P}\,\underline{D}$$

$$\therefore \quad \underline{A}\,\underline{P} = \underline{P}\,\underline{D}$$

If $\underline{P}$ can be found, then the matrix $\underline{A}$ can be diagonalized.

$$\underline{D} = \begin{bmatrix} d_{11} & & & 0 \\ & d_{22} & & \\ & & \ddots & \\ 0 & & & d_{nn} \end{bmatrix}$$ diagonal matrix.

If $\underline{A}$ is a (3×3) matrix and is diagonalized then we may write

$$\underline{P} = [\underline{P}_1 \;\; \underline{P}_2 \;\; \underline{P}_3]$$

columns of $\underline{P}$

then we can write

$$\underline{A}\,\underline{P} = \underline{P}\,\underline{D}$$

$$\underline{A}[\underline{P}_1\;\underline{P}_2\;\underline{P}_3] = [\underline{P}_1 \;\; \underline{P}_2 \;\; \underline{P}_3]\begin{bmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{33} \end{bmatrix}$$

$$[\underline{A}\,\underline{P}_1 \;\; \underline{A}\,\underline{P}_2 \;\; \underline{A}\,\underline{P}_3] = [\underline{P}_1 d_{11} \cdot \underline{P}_2 d_{22} \;\; P_3 d_{33}]$$

or

eigen value

$$\underline{A}\,\underline{P}_1 = \underline{P}_1 d_{11} = d_{11}\underline{P}_1$$

eigen vectors.

If $\underline{P}$ is singular then it can not be diagonalized.

THEOROM

if $\underline{A}$ is a $(n \times n)$ matrix and has n linearly independant eigen vectors, then $\underline{A}$ is diagonalizable.

Ex. Diagonalize $\underline{A}$ given by

$$\underline{A} = \begin{bmatrix} -5 & 9 \\ -6 & 10 \end{bmatrix}$$

find eigen values

$$|\underline{A} - \lambda \underline{I}| = \left| \begin{bmatrix} -(5-\lambda) & 9 \\ -6 & (10-\lambda) \end{bmatrix} \right| = \lambda^2 - 5\lambda + 4$$

$$\lambda_1 = 1$$
$$\lambda_2 = 4$$

b/c λ_1 and λ_2 are distinct which indicates that $\underline{A}$ will have a linearly independant eigen vectors $\underline{k}_1$ and $\underline{k}_2$ hence $\underline{A}$ is diagonizable.

$\underline{\lambda_1 = 1}$

$$[\underline{A} - \lambda_1 \underline{I}]\, \underline{k}_1 = 0$$

$$\left\{ \begin{matrix} (-5-1) & 9 \\ -6 & 10-1 \end{matrix} \right] \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = \left[\begin{array}{cc|c} -6 & 9 & 0 \\ -6 & 9 & 0 \end{array} \right]$$

$$-6k_1 + 9k_2 = 0$$
$$9k_2 = 6k_1$$

let $k_1 = 1$

$$\underline{k}_1 = \begin{bmatrix} 1 \\ 2/3 \end{bmatrix}$$

$\underline{\underline{\lambda_2 = 4}}$

$$[\underline{A} - \lambda_2 \underline{I}]\,\underline{k}_2 = \underline{0}$$

$$\begin{bmatrix} -5-4 & 9 \\ -6 & 10-4 \end{bmatrix}\begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -9 & 9 \\ -6 & 6 \end{bmatrix}\begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\therefore k_1 = 1$ $\qquad \underline{k}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$k_2 = 1$

$$\therefore \quad \underline{P} = [\underline{k}_1 \;\; \underline{k}_2] = \begin{bmatrix} 1 & 1 \\ 3/2 & 1 \end{bmatrix}$$

$$\underline{P}^{-1} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$$

$$\underline{P}^{-1}\,\underline{A}\,\underline{P} = \underline{D}$$

Diagonalized matrix.

$$\begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} -5 & 9 \\ -6 & 10 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 3/2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$$